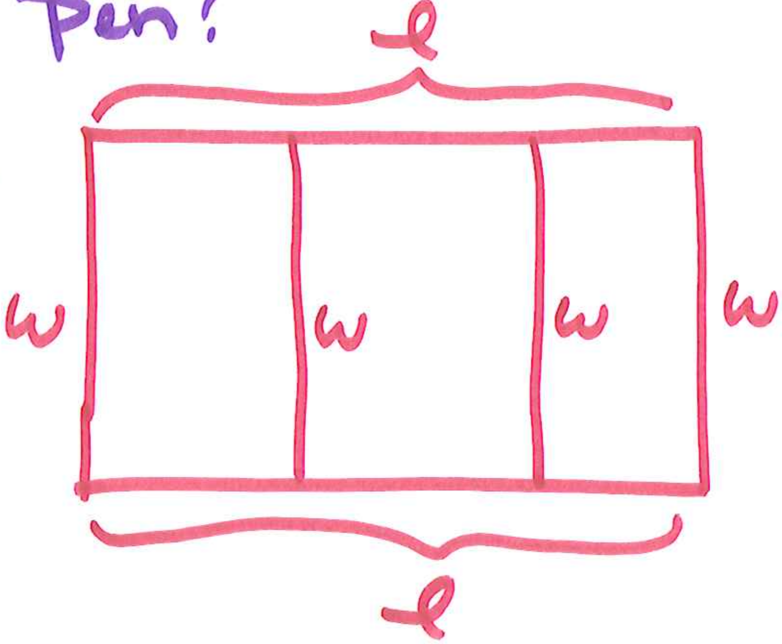
Chapter 7B - Day 2

Ex: A farmer builds a rectangular pen with 3 parallel partitions using 600ft of fence. What dimensions will maximize the total area of the pen?



We know

$$2l + 4w = 600$$
$$2l = 600 - 4w$$
$$l = 300 - 2w$$

We need to maximize area.

$$A = l \cdot w$$
$$= (300 - 2w)w$$
$$= 300w - 2w^2$$

because distances are positive...

$w \geq 0$ $\qquad$ $l \geq 0$

$$300 - 2w \geq 0$$
$$300 \geq 2w$$
$$150 \geq w$$

thus $w \in [0, 150]$

let $A(w) = 300w - 2w^2 = w(300 - 2w)$

so $A'(w) = 300 - 4w$

$$300 - 4w = 0$$
$$300 = 4w$$
$$75\,\cancel{150} = w$$

$A(w)$ is continuous over closed & bounded interval...
use EVT!

$A(0) = 0(300) = 0$

$A(75) = (75)(150) = 11,250$ ← max

$A(150) = (150)(0) = 0$

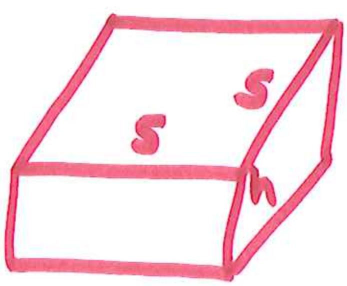
Max area is 11,250 and occurs when

$\boxed{w = 75}$ and $l = 300 - 2w$

$= 300 - 2(75)$

$= \boxed{150 = l}$

Ex: a box is constructed out of 2 different types of metal. The metal for the square top and bottom cost \$7 per square foot and the metal for the sides cost \$14 per square foot. Find the dimensions that minimize cost if the volume of the box is $30 ft^3$.



$$V = s^2 h = 30$$

then $h = \dfrac{30}{s^2}$

We need to minimize cost

$$C = \$7(2s^2) + \$14(4sh)$$
$$= 14s^2 + 56sh$$
$$= 14s^2 + 56s\left(\frac{30}{s^2}\right)$$
$$= 14s^2 + 1680s^{-1}$$

distances are positive....

$s \geq 0$

$h \geq 0$

$\frac{30}{s^2} \geq 0$

true for all $s > 0$

$s \in (0, \infty)$

let $C(s) = 14s^2 + 1680s^{-1}$

then $C'(s) = 28s - 1680s^{-2}$

$= 28s - \frac{1680}{s^2}$

$= \frac{28s^3 - 1680}{s^2}$

$C'(s) = 0$ when $28s^3 - 1680 = 0$

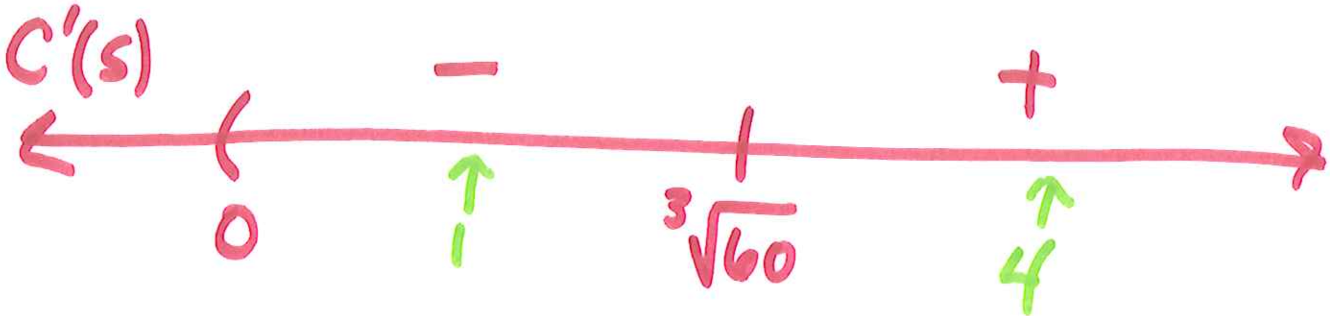
$28(s^3 - 60) = 0$

$s^3 - 60 = 0$

$s^3 = 60$

$s = \sqrt[3]{60} \approx 3.9$

Since s is in an open interval we need to use the 1st derivative test.



$$C'(1) = 28(1) - \frac{1680}{1^2} = -1652 \text{ "−"}$$

$$C'(4) = 28(4) - \frac{1680}{4^2} = 112 - 105 = 7 \text{ "+"}$$

cost is minimized when $\boxed{s = \sqrt[3]{60} \approx 3.9 \text{ ft}}$

and $\boxed{h = \frac{30}{s^2} = \frac{30}{(\sqrt[3]{60})^2} \approx 1.9 \text{ ft}}$